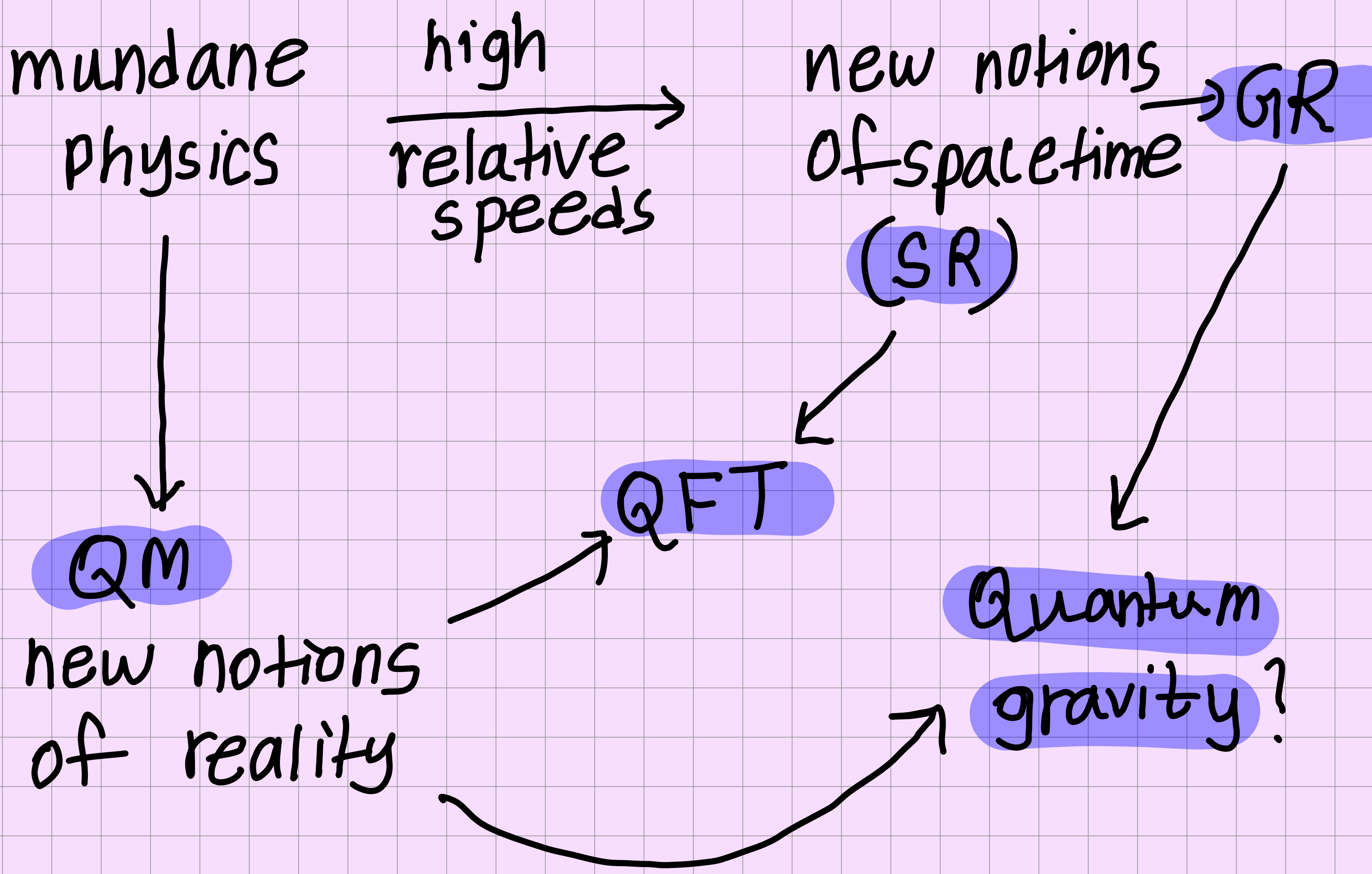
Black hole Information Paradox

by Dr. Suvrat Raju



Why study black-hole information?

quantum gravity features ← teaches us

1) BH & intro to paradox
2) PURE & mixed states
3) holography of information
4) refinement of the paradox

resource!
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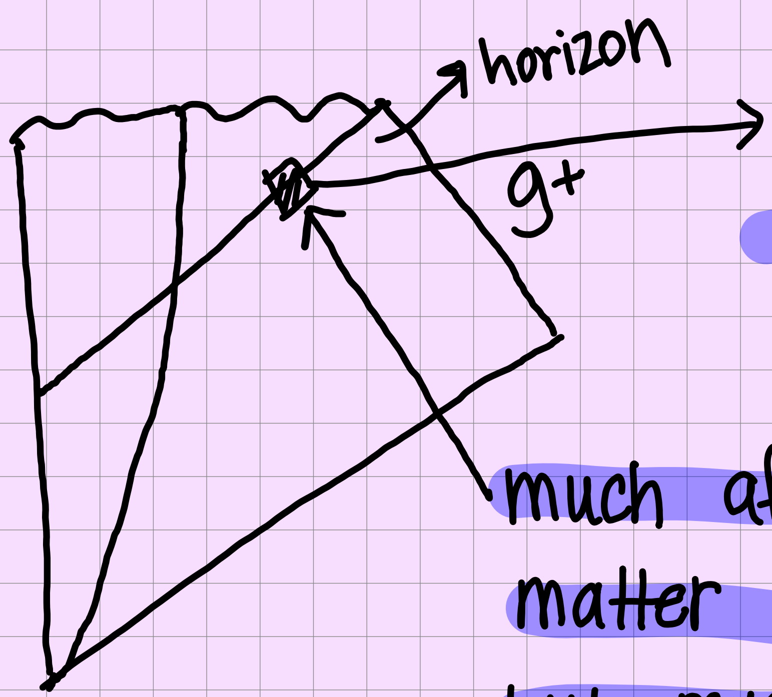
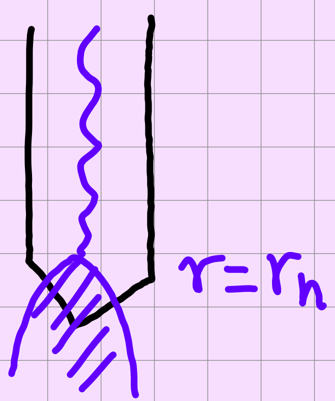


time translational invariance!

much after the infalling matter has fallen in but much before the bh evaporates

Oppenheimer, Sneider
B. datt ⟶ described the geometry



r_h = radius of horizon

for the marked part

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega^2$$

$$f(r) = 1 - \frac{\mu}{r^{d-2}}$$

in $(d+1)$ spacetime dimensions

$$\mu = 8\pi^{(2-d)/2}\,\Gamma\left(\frac{d}{2}\right)\frac{GM}{d-1}$$

$\mu \propto M$

$t \to t + \delta t$ (our ds^2 remains invariant!)

tortoise coordinates

$$dr_* = \frac{dr}{F(r)}$$

as $r \to \infty$, $f(r) \to 1$

$r_* \to \infty$

$f(r) \to 0$ at $r = r_h$,

as $r \to r_h$

$f(r) \to 2K(r - r_h)$

$$K = \frac{f'(r_h)}{2}$$

as $r \to r_h$

$$dr_* \to \frac{dr}{2k(r-r_h)}$$

$$r_* \to \frac{1}{2k}\log[(r-r_h)2k]$$

?

make a coordinate change

(we don't like coordinate singularity)

$$U = -\frac{1}{k}e^{k(r_*-t)}$$

$$V = \frac{1}{k}e^{k(r_*+t)}$$

$$\therefore dU = -(dr_* - dt)e^{k(r_*-t)}$$

$$dV = (dt + dr_*)e^{k(r_*+t)}$$

$$-dUdV = (dr_*^2 - dt^2)e^{2kr_*}$$

$$e^{2kr_*} = 2k(r-r_h) + O(r-r_h)^2$$

now, as $f(r) \to 0$,

$$ds^2 = -dUdV + r^2 d\Omega^2$$

1) At late times
$t \to t + \delta t$ isometry

2) At late times
the horizon is just like empty space for a large black hole

3) near the horizon, wave eqn simplifies and we can identify ingoing and outgoing modes.

4) 2 point function of near horizon modes yields thermal outgoing spectrum

5) thermal rad. causes BH to evaporate to a final state seemingly independent of initial conditions.

consider we are in r_* coordinates

$$(\Box - m^2)\phi = \underset{\sim}{0} \to V(\phi)$$

Box

$$\Box = \frac{1}{\sqrt{-g}} \partial_\mu g^{\mu\nu} \sqrt{-g}\, \partial_\nu$$

$$\sqrt{-g} = f(r)\, r^{d-1} \sqrt{g_r}$$

$$g^{**} = g^{tt} = \frac{1}{f(r)}$$

in r_*, t coordinates
$ds^2 = f(r)(-dt^2 + dr_*^2) + r^2 d\Omega^2$

$$\frac{1}{f(r)} r^{d-1} \partial_* r^{d-1} \partial_* \phi - \frac{1}{f(r)} \partial_t^2 \phi$$

$$+ \frac{1}{r^2} B_\Omega \phi - m^2 \phi = 0$$

$\partial_* =$ derivative w.r.t. r_*

near the horizon, as $r \to r_h$

$\partial_* r = f(r)$

the equation becomes,

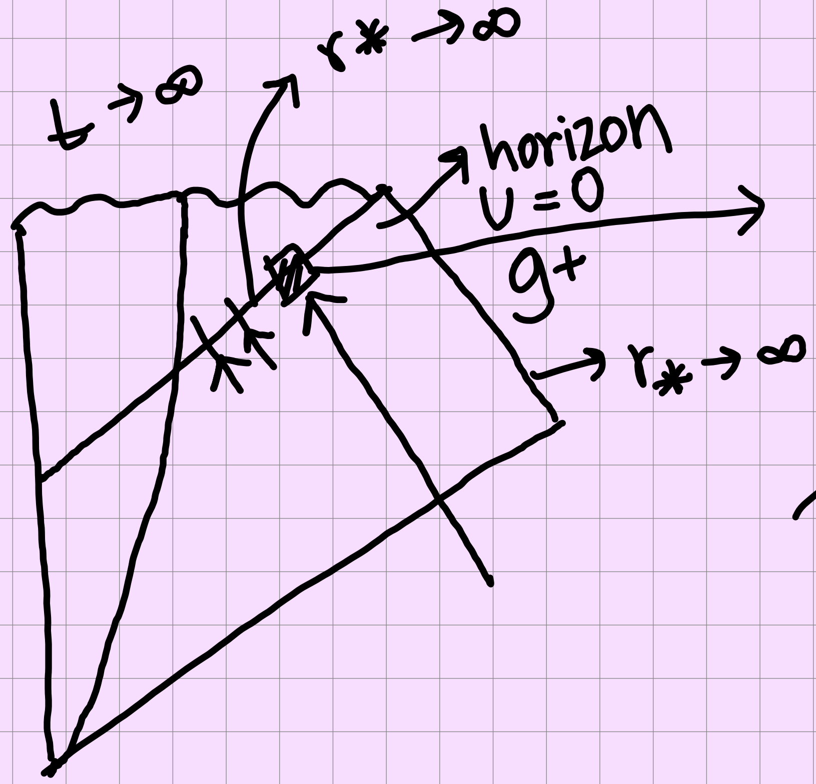
$$\partial_*^2 \phi - \partial_t^2 \phi = 0$$

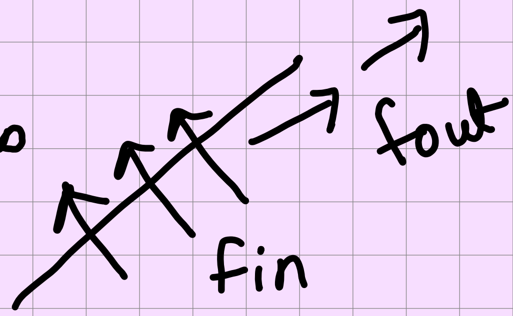
$$\phi = e^{-i\omega t} \cdot y_\ell(\Omega) \begin{cases} e^{-i\omega r_*} \\ e^{i\omega r_*} \end{cases}$$

two possible solutions

$$\phi = e^{-i\omega t} y_\ell(\Omega) e^{-i\omega r_*}$$

$$\phi = e^{-i\omega t} y_\ell(\Omega) e^{i\omega r_*}$$



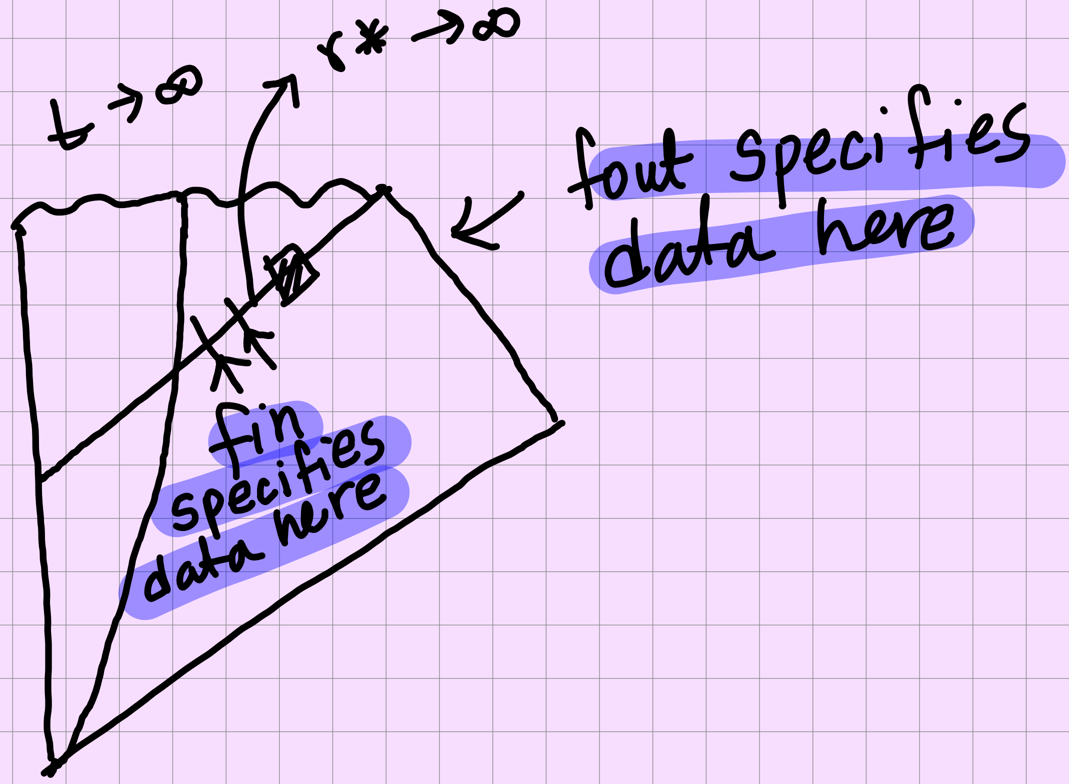


$$\phi = \sum_{\omega,l} e^{-i\omega t} A_{\omega,l} Y_l(\Omega) f_{in}(r_*) + e^{-i\omega t} B_{\omega,l} Y_l(\Omega) f_{out}(r_*) + h.c.$$

$$f_{in}(r_*) \xrightarrow{r_* \to -\infty} h_{\omega,l} e^{-i\omega r_*}$$

$$f_{out}(r_*) \xrightarrow{r_* \to -\infty} g_{\omega,l} e^{-i\omega r_*} + e^{i\omega r_*}$$

$$f_{out}(r_*) \xrightarrow{r_* \to \infty} e^{i\omega r}\, r^{(1-d)/2}$$



$$\phi \sim \sum_{\omega,\ell} B_{\omega,\ell}\left[(-U)^{i\omega/\kappa} + g_{\omega,\ell} V^{i\omega/\kappa} + A_{\omega,\ell} V^{-i\omega/\kappa} h_{\omega,\ell}\right] Y_\ell(\Omega) + \text{h.c.}$$

$$[A_{\omega,\ell}, A^\dagger_{\omega',\ell'}] \sim \delta(\omega-\omega')\delta_{\ell\ell'}$$

$$[B_{\omega,\ell}, B^\dagger_{\omega',\ell'}] \sim \delta(\omega-\omega')\delta_{\ell\ell'}$$

$$[A,B]=0$$

$$[\phi(t, r_*, \Omega), \dot{\phi}(t, r_*', \Omega')] \sim$$

$$i\delta(r_* - r_*')\,\delta(\Omega, \Omega')/\sqrt{g}$$

reference
CLICK!

$$\langle B^{\dagger}_{\omega,\ell} B_{\omega,\ell} \rangle \sim \int \langle \partial_U \phi \, \partial_{U'} \phi \rangle$$

$$(-U)^{i\omega/\kappa} (-U')^{-i\omega'/\kappa} \, dU \, dU'$$

$$\sim \frac{e^{-\beta\omega}}{1 - e^{-\beta\omega}} \, \delta(\omega - \omega') \, \delta_{\ell\ell}$$

when $U - U'$ and $V - V'$ are small,
Ω is close to Ω'.

$$\phi(U, V, \Omega), \; \phi(U', V', \Omega)$$

$$(-U)^{i\omega/\kappa} = e^{i\omega/\kappa \log(-U)}$$

$$\langle \phi(U, V, \Omega) \, \phi(U', V', \Omega) \rangle \sim S^{(1-d)/2}$$

$$\beta = \frac{2\pi}{\kappa}$$

$\kappa \rightarrow$ surface gravity

$$\langle \phi(\vec{x},t)\,\phi(0,0)\rangle \sim \frac{1}{t^2-\vec{x}^2}$$

$$\langle B_{\omega,l}\, B_{\omega,l}^{\dagger} \rangle \sim \frac{1}{1-e^{-\beta\omega}}\,\delta(\omega-\omega')\,\delta_{ll'}$$

$\beta \rightarrow$ temperature

thermal occupancy

the blackhole is radiating

Hawking radiation

the outgoing modes are thermally occupied

$$H = \omega a^{\dagger} a$$

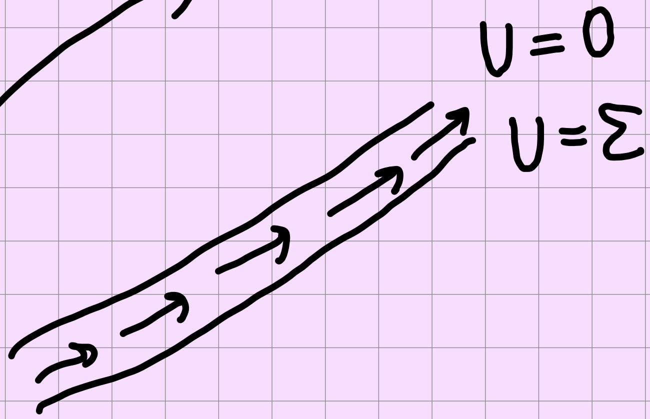
$$\frac{1}{Z(\beta)} \mathrm{Tr}(e^{-\beta H} a^{\dagger} a),\ \frac{1}{Z(\beta)} \mathrm{Tr}(e^{-\beta H} a a^{\dagger})$$

$$Z(\beta) = \mathrm{Tr}(e^{-\beta H})$$

$$\beta \sim \frac{1}{T}$$

$$\langle A_{\omega l}^{\dagger} A_{\omega' l'} \rangle = 0$$

Unruh vaccum



$$T = \frac{\kappa}{2\pi} \qquad \kappa = \frac{f'(r_h)}{2}$$

$$dM = \frac{\kappa \, dA}{8\pi} + \text{work terms}$$

$$dU = T\,dS + \text{work terms}$$

$$S = \frac{A}{4}$$ blackhole entropy

$S \sim 10^{76}$ ← for Solar mass blackhole

$T \sim 10^{-8} K$

hard to detect hawking radiation due to the CMBR

What is the Paradox?!

if the BH is radiating it must be losing some energy.

$$\frac{dM}{dt} = -CAT^{d+1}$$

in 4D,

$$\frac{dM}{dt} = -CAT^{4}$$

d is the spacial dimension

$$f(r) = 1 - \frac{2m}{r}$$

$$f'(r_n) \propto \frac{1}{M}$$

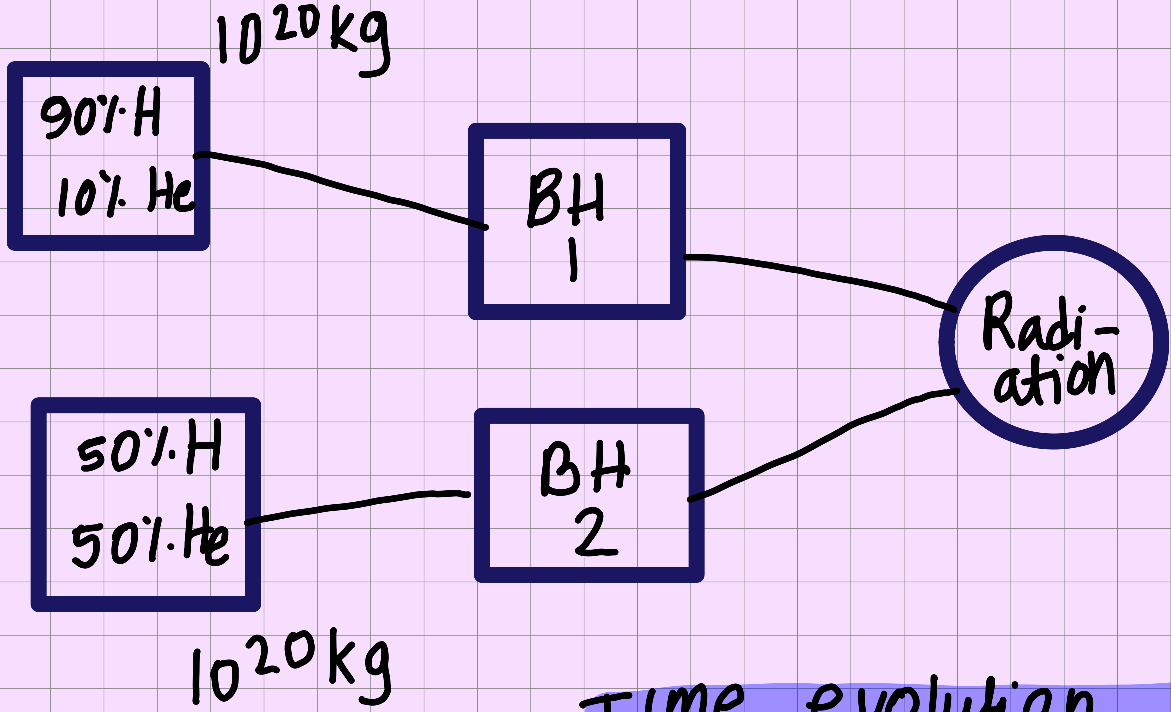
$$T \propto \frac{1}{M} \; ; \; A \propto M^2$$

$$\frac{dM}{dt} \propto \frac{-1}{M^2}$$

in $t \propto M^3$, the BH evaporates completely.

it looks like,

after evaporation, we are left with 'B' excitations which are thermal. → **paradox**



Time evolution is unitary!

but the radiation is same for any two black holes with the same mass. (independent of initial conditions)

$$\rho = \frac{e^{-\beta H}}{Z(\beta)}$$

→ hamiltonian for a SHO

$$Z(\beta) = tr(e^{-\beta H})$$

$$tr(\rho O) = \langle O \rangle_\beta$$

$H = \omega a^\dagger a$

$tr(\rho a^\dagger a)$,

$tr(\rho a a^\dagger)$

assigment: find the traces

how close are pure & mixed states?

$\frac{1}{\sqrt{2}}($ |  $\rangle$ + |  $\rangle)$

$$\frac{1}{\sqrt{2}}(|1\rangle + |0\rangle) = |\psi_{pure}\rangle$$

$$\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix} = \rho_{mixed}$$

$\langle \sigma_3 \rangle_{pure} = 0$ $\sigma_{(x)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\langle \sigma_3 \rangle mixed = 0$

$$\langle \sigma_x \rangle pure = \left(\langle 1|\sigma_x|1\rangle + \langle 0|\sigma_x|0\rangle + \langle 1|\sigma_x|0\rangle + \langle 0|\sigma_x|1\rangle \right) \frac{1}{2}$$

$$\langle O \rangle_{pure} = \langle \psi_{pure} | O | \psi_{pure} \rangle$$

$$\langle O \rangle_{mixed} = tr(\rho_{mixed} O)$$

$$\langle \sigma_x \rangle_{mixed} = 0$$

consider a system with many energy eigenstates in a land of energies ΔE

$(E, E+\Delta E) \longrightarrow e^S$ states

$$P_{micro} = \frac{1}{e^S} \sum_i |E_i\rangle\langle E_i|$$

$$|\psi_{pure}\rangle = \sum a_i |E_i\rangle, \qquad \sum_i |a_i|^2 = 1$$

typical pure state $\langle\langle P \rangle\rangle - tr(P \rho_{micro})$

$$\int \underbrace{\langle \psi_{pure} | P | \psi_{pure} \rangle}_{\text{expectation value of a projector } P} dH = \langle\langle P \rangle\rangle$$

$$dH = \left(\prod_i d^2 a_i\right) \delta\left(\sum_i |a_i|^2 - 1\right) \frac{1}{N}$$

$$N = \frac{2\pi e^S}{\Gamma(e^S)} \qquad \int dH = 1$$

$$\langle \psi_{pure} | = \sum a_j^* \langle E_j |$$

$$\int dH \langle \psi_{pure} | P | \psi_{pure} \rangle = \int \sum_{i,j} a_i a_j^* \langle E_j | P | E_i \rangle dH$$

$$= \int \langle E_i | P | E_i \rangle |a_i|^2 dH$$

$$= \sum_i \langle E_i | P | E_i \rangle \underbrace{\int |a_i|^2 dH}_{\to \frac{1}{e^S}}$$

$$\langle\langle P \rangle\rangle = \sum_{e^S} \langle E_i | P | E_i \rangle$$

$$tr(P \rho_{micro}) = \sum_i \frac{1}{e^S} \langle E_i | P | E_i \rangle$$

$$\int \left(\langle \psi_{pure} | P | \psi_{pure} \rangle - tr(\rho_{micro} P) \right)^2 dH$$

$$\int dH \sum_{\substack{i,j \\ k,l}} \left(\langle E_j | P | E_i \rangle \left(a_i a_j^* - \frac{\delta_{ij}}{e^S} \right) \right) \left(\langle E_l | P | E_k \rangle \left(a_k a_l^* - \frac{\delta_{kl}}{e^S} \right) \right)$$

$i=j, k=l$ or
$i=l, k=j$

$$\int a_i a_j^* a_k a_l^* dH$$

$$= \left(\delta_{ij} \delta_{kl} + \delta_{il} \delta_{jk} \right) \frac{1}{e^S (e^S + 1)}$$

$$\left\langle \sum_{\substack{i,j,\\k,l}} \frac{\delta_{il}\,\delta_{jk}}{e^s(e^s+1)} \right. \langle E_j|P|E_i\rangle\langle E_l|P|E_k\rangle$$

$$= \frac{1}{e^s(e^s+1)} \sum \langle E_j|P|E_i\rangle \underbrace{\langle E_i|P|E_j\rangle}_{P^2}$$

$$< \frac{1}{e^s+1}$$

$$\int dH \sum_{\substack{i,j\\k,l}} \left(\langle E_j|P|E_i\rangle\left(a_i a_j^* - \frac{\delta_{ij}}{e^s}\right)\right)$$

$$\left(\langle E_l|P|E_k\rangle\left(a_k a_l^* - \frac{\delta_{kl}}{e^s}\right)\right)$$

$$< \frac{1}{e^s+1}$$

$$\int \left(\langle \psi_{pure} | P | \psi_{pure} \rangle - tr(\rho_{micro} P) \right)^2 < \frac{1}{e^S + 1}$$

$$\langle \psi_{pure} | P | \psi_{pure} \rangle = tr(\rho_{micro} P) + O(e^{-S/2})$$

1) At late times
$t \to t + \delta t$ isometry emerges

2) The horizon is just like empty Space for a large black hole

3) $\langle B^{\dagger}_{\omega, \ell} B_{\omega' \ell'} \rangle$ $\beta = 1/T$, $T = \frac{K}{2\pi}$

$$= \frac{e^{-\beta\omega}}{1 - e^{-\beta\omega}} \delta(\omega - \omega') \delta_{\ell\ell'}$$

4) Leads to seeming paradox

5) typical pure & microcanonical mixed states are the same for any observation up to $O(e^{-S/2})$

kinematic result

- von neumann entropy can differentiate mixed & pure states

not directly an observable

$$tr(\rho \log \rho) \neq tr(\rho A)$$

6) this calculation (Hawking) is not precise enough to lead to a paradox

small correction theorem

$$\frac{1}{\sqrt{2}}(|0\rangle|1\rangle + |1\rangle|0\rangle) \otimes (|0\rangle|1\rangle + |1\rangle|0\rangle) \dots \text{ N times}$$

$$\downarrow + \Sigma_1 |0\rangle|0\rangle + \Sigma_2 |0\rangle|1\rangle + \Sigma_3 |1\rangle|0\rangle + \Sigma_4 |1\rangle|1\rangle$$

$$\rho = \begin{pmatrix} 1/2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \otimes \begin{pmatrix} 1/2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \otimes \begin{pmatrix} 1/2 & 0 \\ 0 & \frac{1}{2} \end{pmatrix} \dots \text{ N times}$$

small corrections ≠

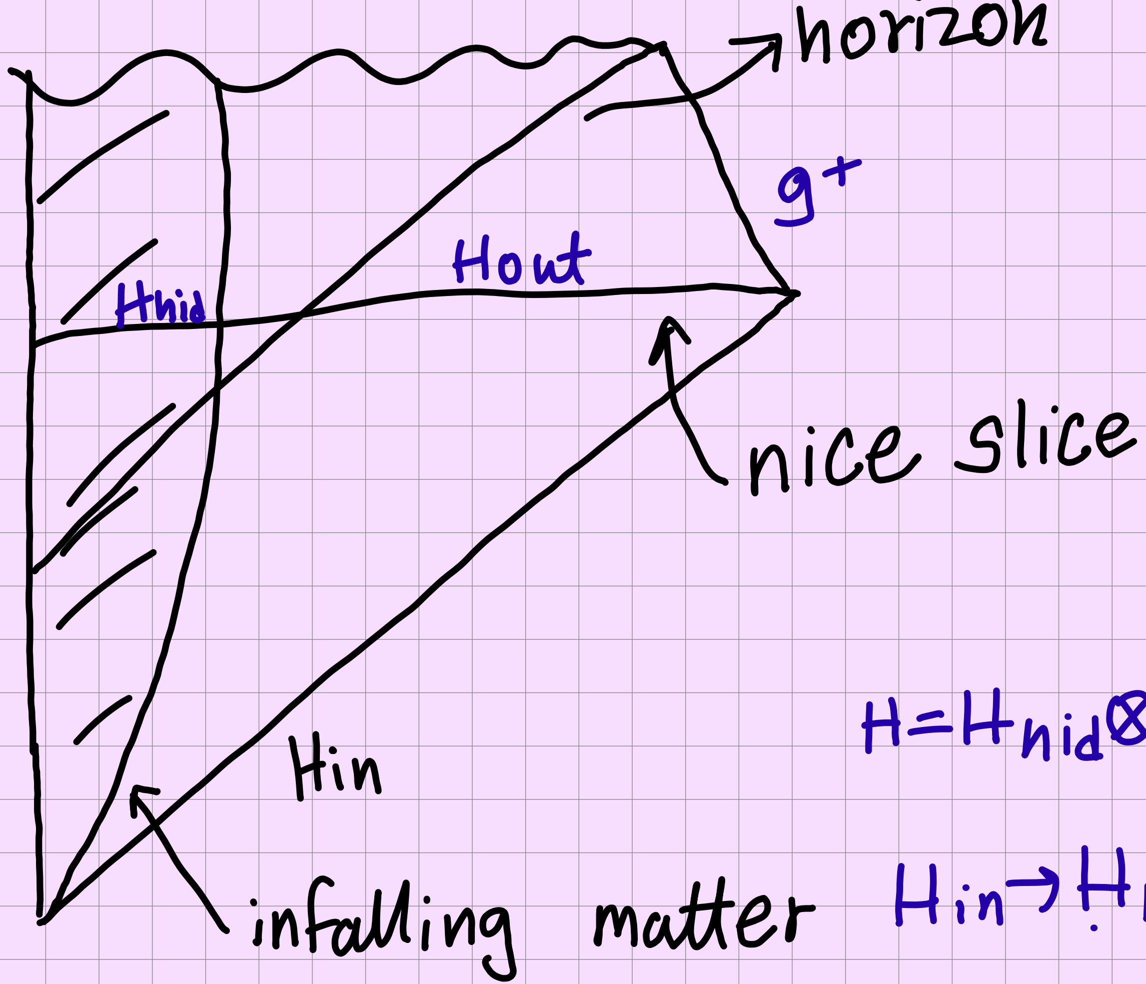
$|\psi_{pure}\rangle$ is close to ρ_{micro} as a state

$$\mathrm{tr}\left(\left(\rho_{micro} - |\psi_{pure}\rangle\langle\psi_{pure}|\right)^2\right) = O(1)$$

7) Typical observations are "close" but the states are not

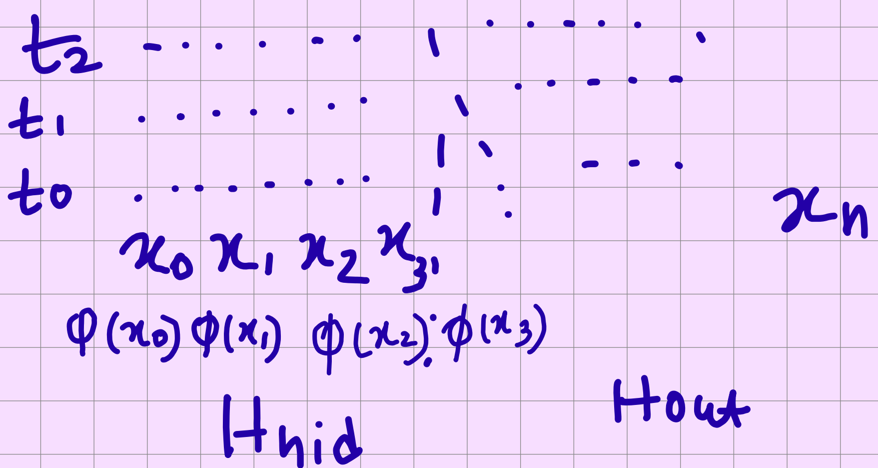
hawking's sophisticated argument

Hawking's paper



$$H = H_{hid} \otimes H_{out}$$

$$H_{in} \to H_{hid} \otimes H_{out}$$



$\rho_{out} = tr(|\psi\rangle\langle\psi|)$

will be mixed $\mathcal{H}_{hid}(M,Q,L)$

"principle of ignorance"

observer outside only knows
M, Q, L inside.

[many configurations inside
which give the same
observables outside]

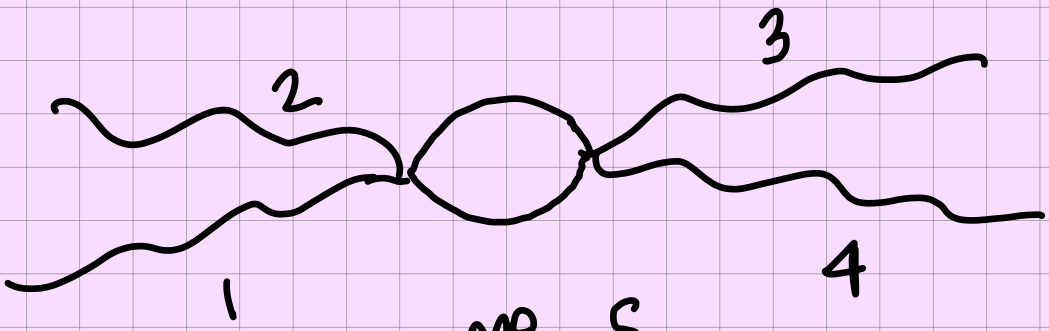
$H = H_{hid} \otimes H_{out}$

$H_{in} \to H_{hid} \otimes H_{out}$

$H_{in} \to H_{out}$ not unitary

pure state   mixed state

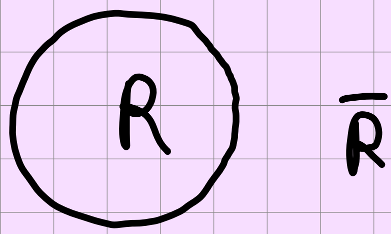
8) principle of ignorance, assumption of factorization suggests state outside is mixed



$e^{-MR} \; e^{S}$

In gravity, principle of holography of information

$\langle\psi| A_{\bar{R}} |\psi\rangle = \langle\psi| U_R^{\dagger} A_{\bar{R}} U_R |\psi\rangle$



observables in the complement of a bounded region are sufficient to specify the state completely.

$\dot{x}_0 \; \dot{x}_1 \,|\, \dot{x}_2 \; \dot{x}_3 \; \dot{x}_4 \,|\, \dots \dots \; \dot{x}_n$

$\phi_1 \qquad \phi(x_2)\,\phi(x_3)$

$\phi(x_4)$

$$\int \vec{g} \cdot \hat{n}\, dA = M$$

Physical intuition

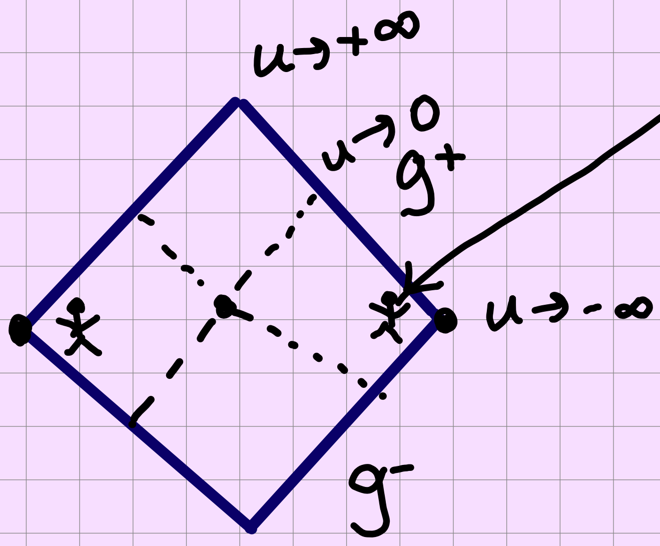
1) Gauss's law
2) Uncertainty principle

↓

Operators that keep M fixed fail to commute with some other operators at ∞.

Birkhoff's theorem

→ **suggest that fixing energy + other observables at ∞ fixes the state.**



observers need to determine the profile of the excitation

Let $\phi(u, r, \Omega)$ be a massless field.

$$u = t - r$$

take $r \to \infty$ while keeping u fixed

$$\phi(u, r, \Omega) \xrightarrow[r \to \infty]{} \frac{1}{r} \underbrace{O(u, \Omega)}$$

intrinstic to g^+
scri
aymptotic observables

important result:

$$\langle \partial_{u'} \mathcal{O}(u', \Omega') \, \mathcal{O}(u, \Omega) \rangle$$

$$= \frac{-1}{4\pi} \frac{1}{u' - u - i\varepsilon} \delta^2(\Omega, \Omega')$$

$$|\psi\rangle = e^{i\lambda \int f(u,\Omega) \mathcal{O}(u,\Omega)} |0\rangle$$

where $f(u,\Omega)$ has support for $u \in (0,1)$

working to $O(\lambda)$ using correlators in $u' \in (-\infty, -\frac{1}{\varepsilon})$

determine $f(u,\Omega)$

$\langle \psi | H \mathcal{O}(u'.\Omega') | \psi \rangle$ to $O(\lambda)$

$$\langle 0| e^{-i\lambda \int f(u'',\Omega'')\, O(u'',\Omega'')\, du''\, d\Omega''} |0\rangle$$

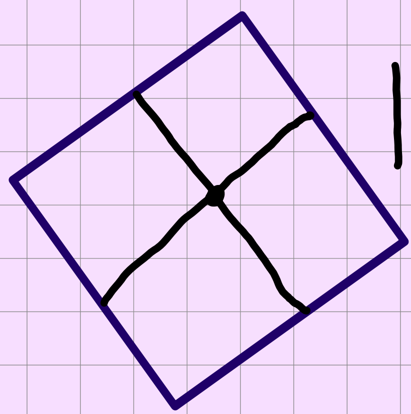
$$\boxed{O(u'',\Omega'')H = H\,O(u'',\Omega'') + \frac{i\partial}{\partial u''} O(u'',\Omega'')}$$

$$= \frac{\lambda}{4\pi} \int \frac{f(u'',\Omega')}{u''-u'+i\varepsilon} du'' + O(\lambda^2)$$

$$= -\frac{\lambda}{4\pi} \sum_n \int f(u'',\Omega') \frac{(u'')^n}{(u')^{n+1}} du''$$

$$= \sum_{n=0}^{\infty} -\frac{\lambda}{4\pi (u')^{n+1}} \int (u'')^n f(u'',\Omega')$$

9) observables in complement of a bounded region determine the state for pure states.



$$|\psi\rangle = e^{i\int f(x,\Omega) O(x,\Omega)\, dx\, d\Omega} |0\rangle$$

$$\langle\psi| H O(u,\Omega) |\psi\rangle \quad , u \in \left(-\infty, -\frac{1}{\varepsilon}\right)$$

$$\sim \int_0^1 \frac{f(x,\Omega)}{x-u} dx$$

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle \right)$$

$$U_1 |\psi\rangle$$

$$\langle\psi| U_1^\dagger A_2 U_1 |\psi\rangle = \langle\psi| A_2 |\psi\rangle$$

$\phi, \tilde{\phi}$

related symmetry

$\phi \xrightarrow[r\to\infty]{} \frac{1}{r} O, \quad \tilde{\phi} \xrightarrow[r\to\infty]{} \frac{1}{r}\tilde{O}$

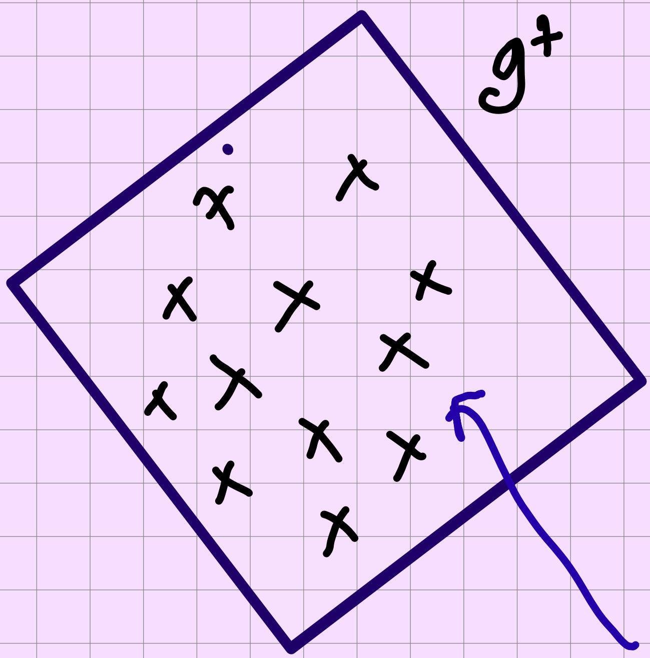
$$|\tilde{\Psi}\rangle = e^{i\lambda \int f(x,\Omega)\,\tilde{O}(x,\Omega)\, dx\, d\Omega} |0\rangle$$

$$\langle\Psi| H\tilde{O} |\Psi\rangle = 0 + O(\lambda^2)$$

$$\langle\tilde{\Psi}| H\tilde{O} |\tilde{\Psi}\rangle = \langle\Psi| HO |\Psi\rangle = \int_0^1 \frac{f(x,\Omega)\,dx}{x-u}$$

$$\langle\tilde{\Psi}| HO |\tilde{\Psi}\rangle = 0 + O(\lambda^2)$$

g^+



$A(g^+)$ – algebra of operators of g^+.

bulk is unknown.

asymptotically flat spacetime

$A_{-\infty}$ - algebra of operators for $u \in (-\infty, -\frac{1}{2})$

every element of $A(g^+)$ can be approximated arbitrarily well by an element of $A_{-\infty}$

Assumptions:

1) these algebras continue to make sense.

$$A(g^+) = \text{Span}\{O(u_1, \Omega_1), \ldots\ldots O(u_n, \Omega_n) \ldots\}$$

this also includes metric fluctuations.

$$\phi(r,u,\Omega) \xrightarrow[r\to-\infty]{} \frac{1}{r} O(u,\Omega)$$

flat space

$$ds^2 \xrightarrow[r\to\infty]{} -du^2 - 2du\,dr + r^2 d\Omega^A d\Omega^B \gamma_{AB} +$$

$$\frac{2m}{r} du^2 + r C_{AB} d\Omega^A d\Omega^B + D^B C_{AB} du\, d\Omega^A$$

mass aspect

$$N_{AB} = \partial_u C_{AB}$$

news — sheer

$$A_{-\infty} = \text{span}\{ O(u_1) \dots O(u_n)$$

$$C_{AB}(u_1') \dots C_{AB}(u_m')$$

$$m(u_1'') \dots m(u_m'')\}$$

$$u_i \in (-\infty, -\tfrac{1}{\varepsilon})$$

2) hilbert space:

energy is positive

$$H = A(g^+)|0\rangle$$

$\forall |n\rangle \in H$, $\exists X_n \in A_{-\infty}$

such that $X_n|0\rangle = |n\rangle$

has nothing to do with gravity

Reeh-Schleider theorem

consider

$$\int_{-\infty}^{\infty} O(u) f(u) |0\rangle = |f\rangle$$

we can find that $\int_{-\infty}^{-1/\varepsilon} du\, O(u) g(u) |0\rangle = |f\rangle$

so that

$$\| |f\rangle - |g\rangle \| \approx 0$$

Proof by Contradiction

say $\exists f$ such that $\langle f | O(u) | 0 \rangle = 0$
$\forall u \in (-\infty, -\frac{1}{\epsilon})$

$$= \sum_E \langle f | E \rangle \langle E | O(u) | 0 \rangle$$

$$= \sum_E \langle f | E \rangle \langle E | O(0) | 0 \rangle e^{iEu}$$

It is analytic when u is extended in the upper half plane

$\Rightarrow \langle f | O(u) | 0 \rangle = 0 \; \forall$ real u.
which is absurd.

$\therefore$ no such $|f\rangle$ exists. ∎

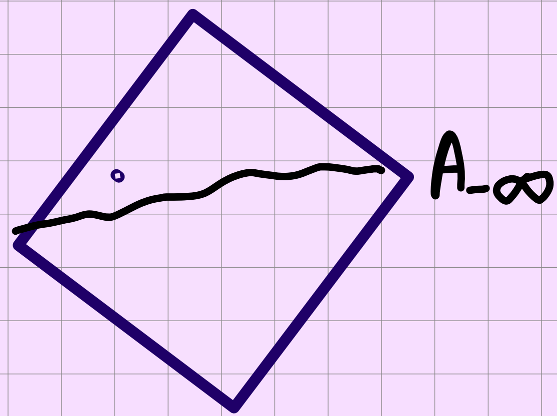
edge-of-wedge theorem

PCT, spin statistics and all that.

} further reading

3) $\hat{H} \in A_{-\infty} = \frac{\#}{G_N} \lim_{u \to -\infty} \int m(u, \Omega) d^2\Omega$

$\hat{H} \in A_{-\infty}$ ← requires gravity



Assumption:

this remains true in the UV-complete theorem.

gauss's law?

$P_0 = |0\rangle\langle 0| \in A_{-\infty}$

* $\hat{H}$ = hamiltonian
* H = hilbert space

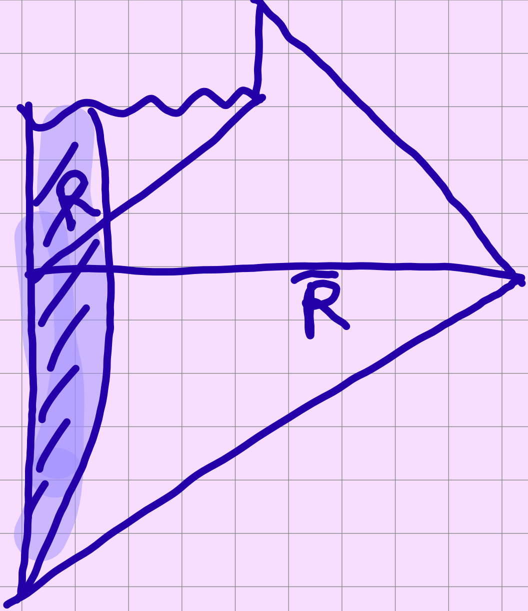
$$T = \sum_{n,m} C_{nm} |n\rangle\langle m|$$

$$= \sum_{n,m} C_{nm} X_n |0\rangle\langle 0| X_m^\dagger$$

$$X_n, X_m \in A_{-\infty} = \sum_{n,m} \underline{C_{nm} X_n P_0 X_m^\dagger}$$

sum of product of 3 operators from A_-

10) All elements of $A(g^+)$ can be approximated arbitrarily well in $A_{-\infty}$.



$H = H_R \otimes H_{\bar{R}}$

wrong assumption

1) $H \neq H_R \otimes H_{\bar{R}}$ when R is bounded and $\bar{R}$ is its compliment

2) If we coarse grain observation, hilbert space might factorise effectively.

Page curve

$H = H_m \otimes H_n$

$\dim(H_m) = m$
$\dim(H_n) = n$
assume that $m < n$

Consider "generic" state

$$|\psi\rangle = \sum_{j=1}^{n} \sum_{i=1}^{m} A_{ij} |i\rangle |j\rangle$$

by a change of basis

$$A = m \left\{ \underbrace{\begin{pmatrix} \tilde{A}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \tilde{A}_{22} & & & & & & & \\ 0 & 0 & \ddots & & 0 & 0 & 0 & 0 & \\ 0 & 0 & & \ddots & & & & & \\ & & & & \ddots & & & & \\ & & & & & & \tilde{A}_{mm} & 0 & 0 \end{pmatrix}}_{n} \right.$$

$$\text{eigen}(P_m) = \left(|\tilde{A}_{11}|^2, |\tilde{A}_{22}|^2 \ldots |\tilde{A}_{mm}|^2 \right)$$

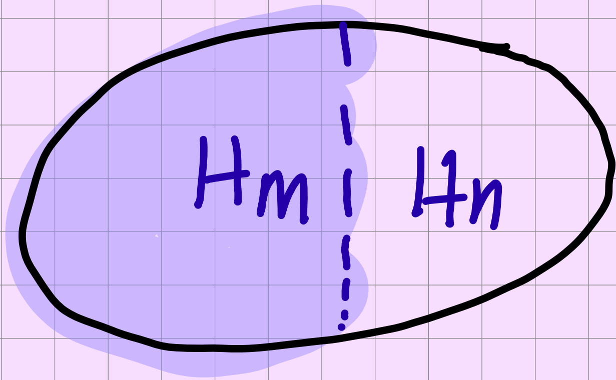
$$S_m = -\text{tr}(P_m \log P_m)$$

$$= -\sum_{i} |\tilde{A}_{ii}|^2 \log(|\tilde{A}_{ii}|^2)$$

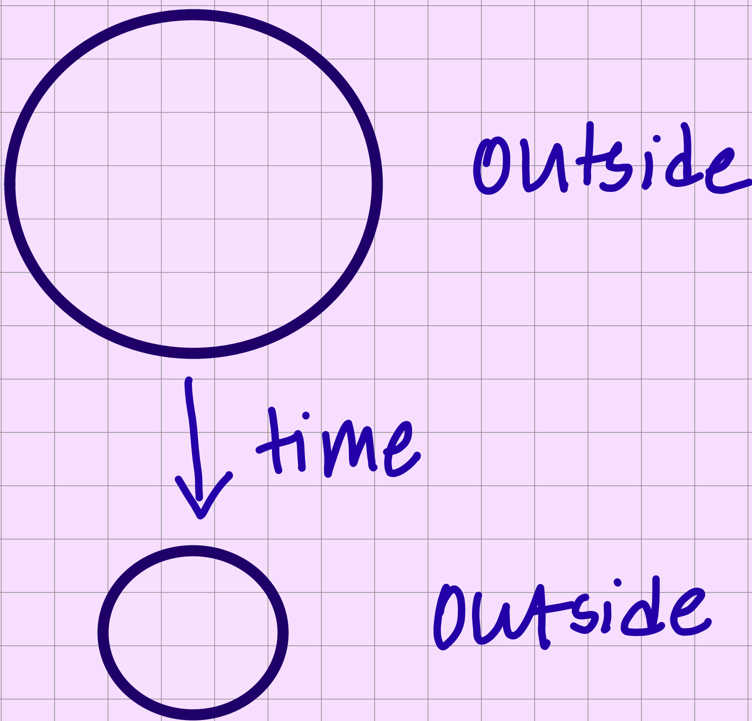
$$= \sum_{i=1}^{m} \frac{1}{m} \log m = \log m$$

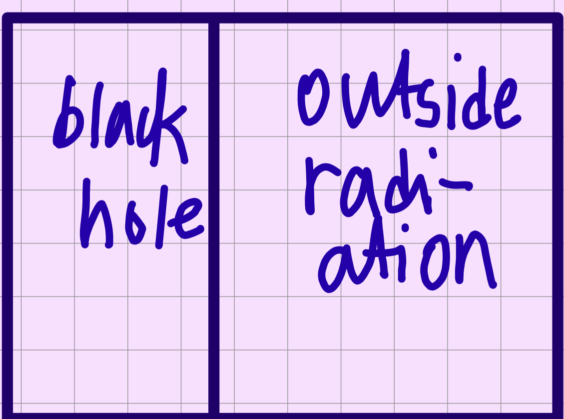
$$\boxed{\sum_{i=1}^{m} |\tilde{A}_{ii}|^2 = 1}$$

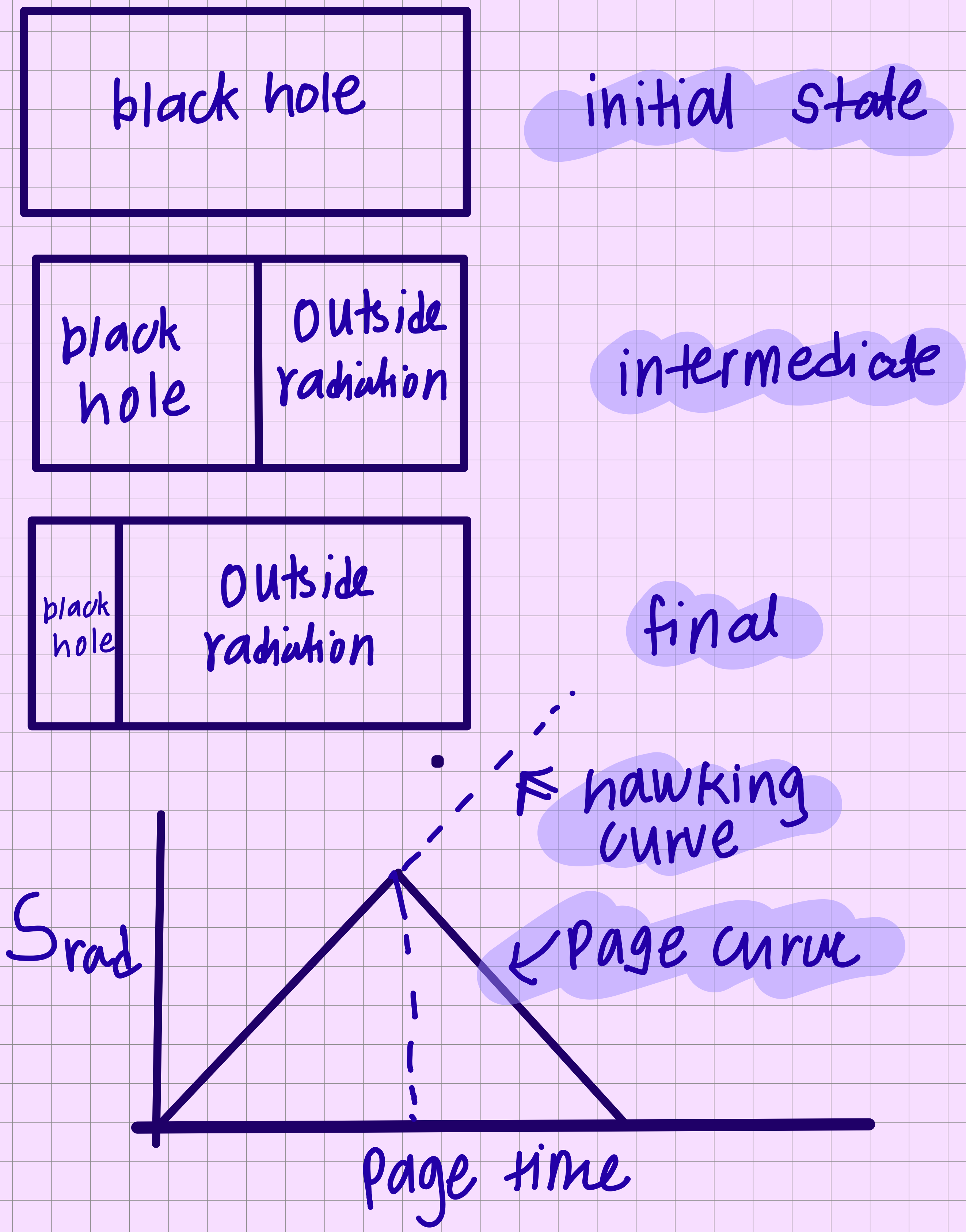
$$\langle S_m \rangle = \log(\min(m,n))$$



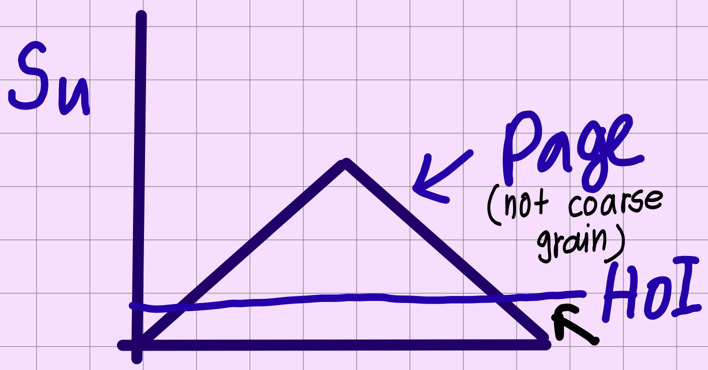
Page's argument:

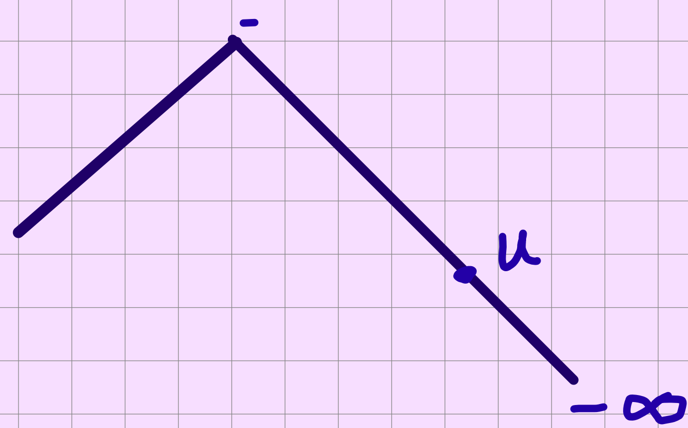





black hole
initial state
black hole
outside radiation
intermediate
black hole
outside radiation
final
hawking curve
page curve
Srad
page time

Page argument also assumes factorization of Hilbert space into inside & outside



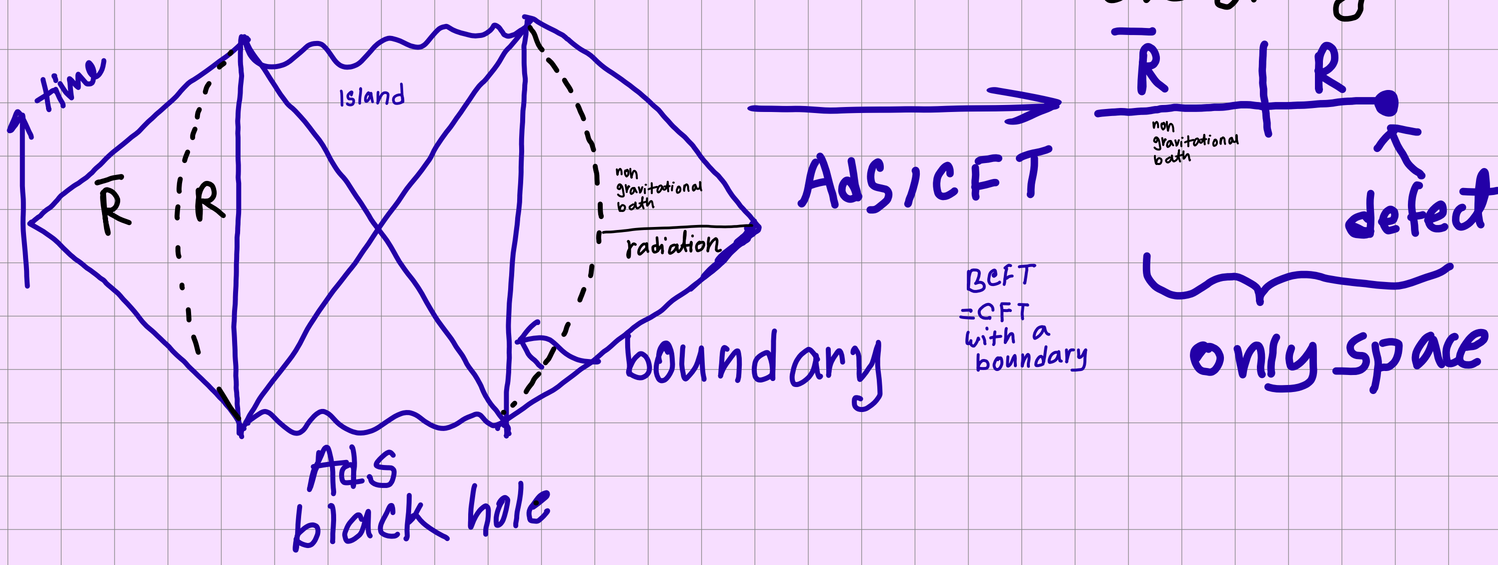


11) $\langle S_m \rangle = \log(\min(m,n))$

12) $H \neq H_R \otimes H_{\bar{R}}$

$$S_{fine} = -tr(\rho \log \rho)$$

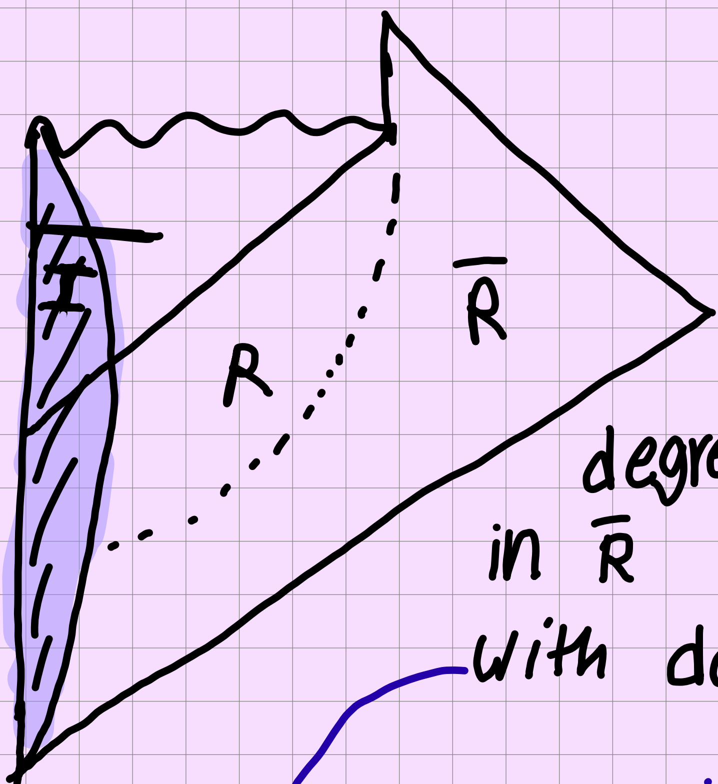
fine-grained von-Neumann entropy where you keep track of everything



13) AdS black hole coupled to non-gravitational system with a boundary defect.

In this setting, we can compute the page curve of part of a bath.

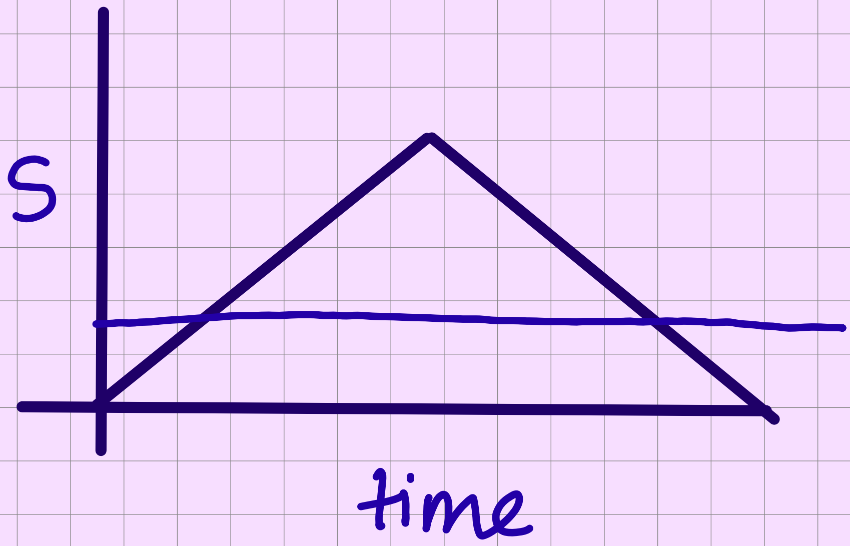
$$\text{ext}\left(\frac{A I}{4G} + S_{QFT}\left(\text{rad}^n \cup \text{Island}\right)\right)$$

14) 

degrees of freedom in $\bar{R}$ are redundant with dof in I.

weak gravity ≠ no gravity

all are redundant with all dof



notes by nazlee
(or nafisa)