# String Cosmology

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Plan:

1) String compactifications
2) Moduli stabilization
3) String inflation — a model
4) Axions & dark energy from string theory

Superstring theory lives in 10D but we see only 4D
$\Rightarrow$ we need to study string compactification

$$\mathbb{R}^{1,9} \longrightarrow X_{10D} = \mathbb{R}^{1,3} \times Y_6$$

with $\dfrac{1}{\mathrm{Vol}(Y_{6D})^{1/6}} = M_{KK} \gg E_{LHC} \sim O(1)\,\mathrm{TeV}$

Idea

$E$

$M_p$

10D String theory (Type IIB)

$M_s$

10D supergravity — considering only massless string modes

$M_{KK}$

4D supergravity

$M_p \sim 10^{18}\,\text{GeV}$

$M_s = \dfrac{1}{l_s}, \; l_s = 2\pi\sqrt{\alpha'}$

$M_{KK} \simeq \dfrac{M_s}{\mathcal{V}^{1/6}} \simeq g_s \dfrac{M_p}{\mathcal{V}^{2/3}}$

Trust EFT if $g_s \ll 1$ and $\mathcal{V} \gg 1$

$\boxed{M_{KK} \ll M_s \ll M_p}$

$N = 8$ if $Y_6 = T^6$

$\downarrow$

$N = 2$ if $Y_6 = CY_3$

$\searrow$

$N = 1$ ORIENTIFOLD projections

$\alpha = 1, 2$ Weyl spinor

$Q^A_\alpha$

$\bar{Q}^A_{\dot\alpha}$

$\{Q_\alpha, \bar{Q}_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} P_\mu$

$N = 1$

$A = 1, \ldots, N$

$\sigma^\mu = (\mathbb{1}, \vec{\sigma})$

$e^{i a^\mu P_\mu + i \epsilon(x) Q_\alpha}$

$$S_{10D} \supset M_s^8 \int d^{10}x \sqrt{-g_{10D}}\, e^{-2\phi} R_{10D}$$

$\downarrow$ Dimensional reduction

$$M_s^8 \left( \int dy^6 \sqrt{-g_{6D}} \right) \int d^4x\, e^{-2\langle\phi\rangle} \sqrt{-g_{4D}}\, R_{4D}$$

$\Downarrow$

$\mathrm{Vol}(y_{6D})$

$$M_s^8\, \mathrm{Vol}(y_{6D})\, e^{-2\langle\phi\rangle} \int d^4x \sqrt{-g_{4D}}\, R_{4D}$$

$$\mathrm{Vol}(y_{6D}) = \mathcal{V}\, l_s^6 = \mathcal{V}\, M_s^{-6}$$

$$M_s \frac{\mathcal{V}}{g_s^2} \simeq M_p^2$$

$e^{\langle\phi\rangle} = g_s$

string coupling

We focus on string compactification which leads to $N=1$ SUSY in 4D since

1) $N=1$ SUSY gives control over EFT due to non-renorm. theorems.

   $W$ receives only non-pert. corrections.

2) $N=1$ is chiral $\Rightarrow$ can get standard model

3) $N=1$, SUSY should be broken spontaneously at low energies via a dynamical mech. to recover good prop.

4) soft terms for gauginos and squarks & sleptons are generated naturally in supergravity via gravitational interactions.

# Massless Spectra

| | Type IIB |
|---|---|
| NS-NS | $G_{MN}$, $B_{[MN]}$, $\varphi$ |
| R-R | $C_0$, $C_2$, $C_4$ |
| NS-R<br>R-NS | $\psi^1_{M,+}$ $\psi^2_{M,+}$, $\lambda^1_-$, $\lambda^2_-$ |

$M, N = 0, 1, \ldots, 9$

$\mu, \nu = 0, 1 \ldots 3$

$m, n = 4, \ldots, 9$

$G_{\mu m}$

$C_{\mu m n p}$

## heterotic

BOSONS : $G_{MN}, B_2, \varphi, A^a_M$

$$A^a_M \longrightarrow A_\mu$$

$a = 1, \ldots, \dim(G)$

$G = SO(32)$ , $E_8 \times E_8$

$$E_8 \supset E_6 \supset SO(10) \supset SU(5) \supset SU(3) \times SU(2) \times U(1)$$

GUT

heterotic Phenomenology has issues

i) $\alpha_{GUT} = \frac{g^2_{GUT}}{4\pi} = \frac{g_s^2}{\mathcal{V}} \propto \frac{1}{2S}$

hard to trust EFT with $g_s << 1$
$\mathcal{V} >> 1$

ii) moduli stabilization

Dim reduction of $G_{MN}(x)$ gives rise to 4D to many scalars.

1) Kähler moduli (parametrise deformations of $Y_{6D}$ in size)

e.g. $v(x^\mu)$

$T_i$ $\quad i = 1, \ldots, h^{1,1}(Y_{6D})$

harmonic (1,1)-forms

2) complex structure moduli (deformations in shape)

$U_\alpha$

+ Axio dilatons

$S = e^{-\phi} + i C_0$ $\quad \alpha = 1, \ldots, h^{1,2}(Y_{6D})$

$\langle R_e(S) \rangle = \frac{1}{g_s}$

harmonic (1,2)-forms

in general

$$h^{1,1}(Y_{6D}) \sim \mathcal{O}(100) \sim h^{1,2}(Y_{6D})$$

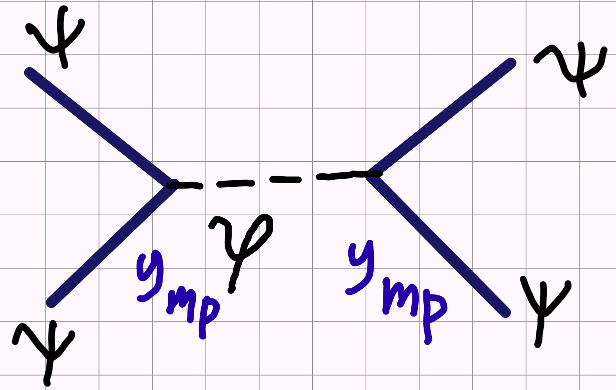
Moduli are uncharged 4D scalars with gravitational couplings to matter which are massless at leading order in supergravity EFT

$\Rightarrow$ Pheno Disaster for two reasons

i) they would mediate Yukawa-like long range 5-TH FORCES which are not observed.

$$m_{mod} \gtrsim 1 meV$$

$$\mathcal{L}_{4D} \supset \frac{m_\psi^2}{m_p} \varphi \bar{\psi} \psi$$



## ii) Lack of Predictability

all features of 4D EFT should be determined dynamically by VEV of a modulus $\varphi$

$$g_{ym} = g_{ym}(\varphi) \qquad y_{i,k} = y_{i,k}(\varphi)$$

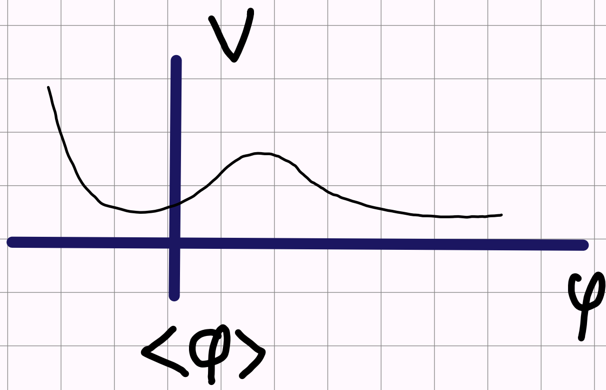
$$m_i = m_i(\varphi)$$

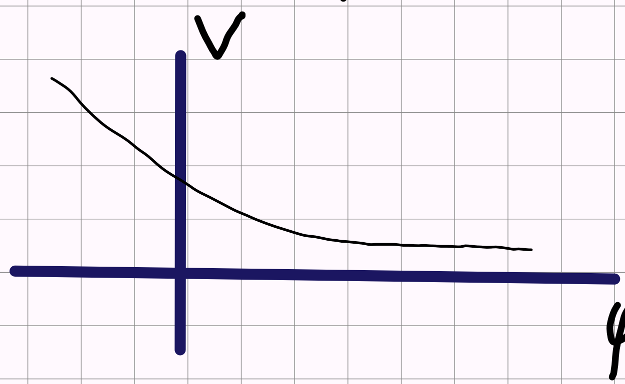
$$M_{susy}(\varphi) \qquad M_s(\varphi) \qquad \Lambda(\varphi)$$

$$H_{inf}(\varphi) \qquad M_{KK}(\varphi)$$

need to introduce effects beyond leading order to generate a potential for $\varphi$ with a stable minimum

"moduli stabilization"




$V(\varphi_i) \neq 0$

$\partial_{\varphi_i} V = 0 \quad \forall \varphi_i$

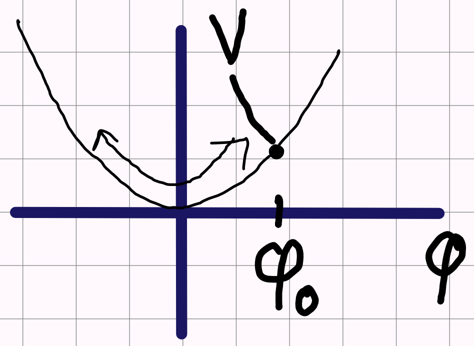
Post definite

$\partial^2_{\varphi_i \varphi_j} V$

eigenvalues give $m_i^2$

# cosmological moduli problem

$$V = \frac{1}{2} m^2 \phi^2$$




## During inflation

$$V = \frac{1}{2} m^2 \varphi^2 + \frac{1}{2} H_{inf}^2 (\varphi - \varphi_0)^2$$

$$\simeq \frac{1}{2} H_{inf}^2 (\varphi - \varphi_0)^2$$

$m << H_{inf}$

$$\ddot{\varphi} + 3H\dot{\varphi} - m^2\varphi = 0$$

$H_{osc} \sim m$ $\varphi$ starts oscillating and stores ENERGY

$$\rho_\varphi^{osc} \sim m^2 \varphi_0^2 \sim m_s^2 M_p^2$$

$$\rho_r^{osc} \sim H_{osc}^2 M_p^2 \sim m^2 M_p^2$$

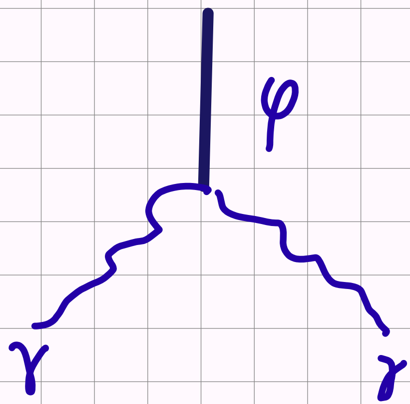
$H_{osc} \sim m$

But $\varphi$ quickly comes to dominate since

$$\rho_\varphi \propto a^{-3} \qquad \rho_r \propto a$$

## $\varphi$ decays when

$$H_{dec} \sim \Gamma_{\varphi \to \gamma\gamma} \sim \frac{1}{M_p^2} m^3$$




$$H_{dec} \sim H_{osc} \left(\frac{m}{M_p}\right)^2 << H_{osc}$$

$$m << M_p$$

$$\rho_\varphi^{dec} = \rho_\gamma^{dec} \left(\frac{a_{dec}}{a_{osc}}\right) = \rho_\gamma^{dec} \left(\frac{M_p}{m}\right)^{4/3} >> \rho_\gamma^{dec}$$

$\Rightarrow$ Late time reheating by $\varphi$ decay

$$T_{rh} > T_{BBN} \sim O(1)\text{ MeV}$$

$$H_{dec} \sim \frac{T_{rh}^2}{M_p} \sim \Gamma \quad \Rightarrow T_{rh} \simeq \sqrt{\Gamma M_p} = m_\varphi \sqrt{\frac{m_\varphi}{M_p}}$$

$$> O(1)\text{ MeV}$$

$$\Rightarrow m_\varphi > O(10)\text{ TeV}$$

hard to get thermal neutrino DM with $m_{DM} \sim O(1)$ TeV

$$\Rightarrow T_f \sim \frac{m_{DM}}{10} \sim 10^2\text{ GeV}$$

FREEZE OUT

to avoid CMP $m_\varphi \gtrsim O(100)$ TeV

but

$m_{DM} \sim m_{3/2} \sim m_\varphi \Rightarrow$ DM OVERPRODUCTION

susy wimp

gravitino mass

"sequestered susy"

$\Rightarrow$ need

$m_{DM} \ll m_\varphi \sim m_{3/2}$

$\wr$

$O(1)$TeV

$T_{rh} > T_f \sim 10^2$GeV

when mssm is realised with open strings on D3-branes

4D spacetime filling

"Dine Seiberg Problem"

hard to find a minimum where EFT is under control

since

$$V_{tree}(\varphi) = 0$$

$$\Rightarrow V_{quantum}(\varphi) \neq 0$$

3 contributions

$$V_{quantum} = V_{g_s} + V_{\alpha'} + V_{non-pert}$$

i) String Loops

$$V_{g_s} = \sum_{m \geq 1} g_s^m V_{(m)}$$

$$g = \frac{1}{Re(S)} \ll 1$$

ii) higher derivative $\alpha'$ corrections

$$V_{\alpha'} = \sum (\alpha')^m V_{(m)}$$

$$Vol(Y_{6D}) = \mathcal{V} \ell_s^6 = \mathcal{V} (2\pi)^6 (\alpha')^3$$

$$\frac{\alpha'}{(2\pi)^2 \, Vol(y_6)^{1/3}} = \frac{1}{\mathcal{V}^{1/3}} << 1$$

iii) non - pert corr in $g_s$

$$V_{non\text{-}pert} = \sum_{n \geq 1} e^{-\frac{n}{g_s} A_{(n)}} V_{(n)}$$

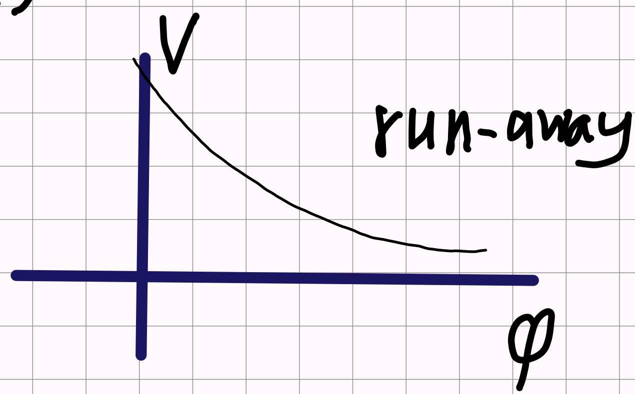
focus on pert. corr.

$$V(\varphi) = \frac{a}{\varphi} - \frac{b}{\varphi^2} + \frac{c}{\varphi^3} + \mathcal{O}\left(\frac{1}{\varphi^4}\right)$$

2 cases

1) trust EFT only for $\varphi >> 1$ and $a \sim b \sim c \sim O(1)$

$$\Rightarrow V(\varphi) = \frac{a}{\varphi}$$



2) use all terms to find a minimum

$$V'(\varphi) = 0 \iff \langle\varphi\rangle_{\pm} = \frac{b}{a}\left(1 \pm \sqrt{1 - \frac{3ac}{b^2}}\right)$$

$$b^2 \geq 3ac$$

set $b^2 > 3ac$

$V''(\langle\varphi\rangle_-) > 0$ min

$V''(\langle\varphi\rangle_+) < 0$ max

$V(\langle\varphi\rangle_-) = 0$ $\quad ac = \frac{b^2}{4}$

$b \gg a$

$$\Rightarrow \langle\varphi\rangle_- = \frac{b}{2a} \gg 1$$

focus on pert corr.

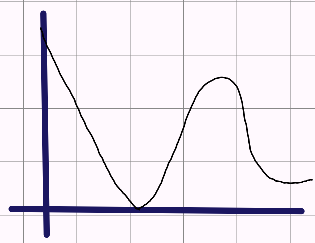
$$V(\langle\varphi\rangle_-) = \underbrace{\frac{a}{\langle\varphi\rangle_-}}_{\frac{2a^2}{b}} - \underbrace{\frac{b}{\langle\varphi\rangle_-^2}}_{\frac{4a^2}{b}} + \underbrace{\frac{c}{\langle\varphi\rangle_-^3}}_{\frac{2a^2}{b}} + \mathcal{O}\left(\frac{1}{\varphi^4}\right)$$

trust EFT?

$c \gg b \gg a$

## way-out

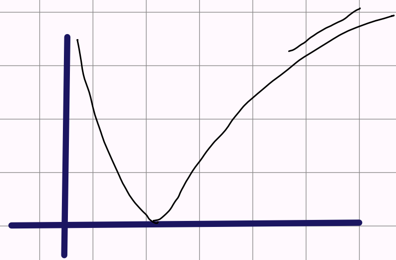
do NOT balance different terms in same exp but diff terms in diff exp



Like,

i) $\alpha'$ corr $\longleftrightarrow$ $g_s$ corr

pert stab



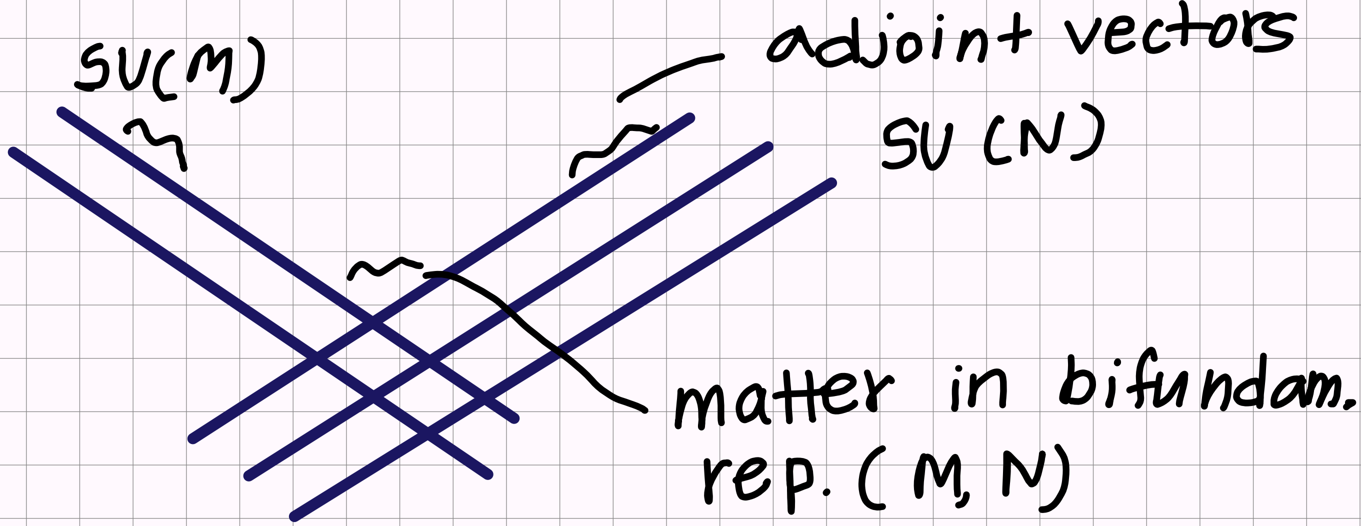
ii) pert corr. $\longleftrightarrow$ non-pert. corr

Type IIB string pheno received a lot of attention after discovery of branes since

1) presence of D-branes provides

i) non-abelian gauge symmetry

ii) chiral matter




2) can turn on background fluxes

$$C_0, C_2, C_4, B_2 \Rightarrow F_1 = dC_0 \quad F_3 = dC_2$$

$$F_5 = dC_4 \quad H_3 = dB_2$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$F_2 = dA_1$$

$$\frac{1}{(2\pi)^2 \alpha'} \int_{\Sigma_3^i} F_3 = m^i \in \mathbb{Z}$$

$\forall i = 1, \dots, 2(h^{1,2}+1)$

3 cycles

$$\frac{1}{(2\pi)^2 \alpha'} \int_{\Sigma_3^i} H_3 = m^i \in \mathbb{Z}$$

$\Rightarrow$ generate a potential for

axio-dilaton and cx str. moduli with small back reaction on CY geometry.

$$ds^2 = e^{2A(y)}\eta_{\mu\nu}dx^\mu dx^\nu + \underbrace{e^{-2A(y)}}_{\text{warp factor}}\underbrace{g_{mn}}_{\text{CY metric}}dy^m dy^n$$

$\Rightarrow$ Fix most of moduli with EFT under control

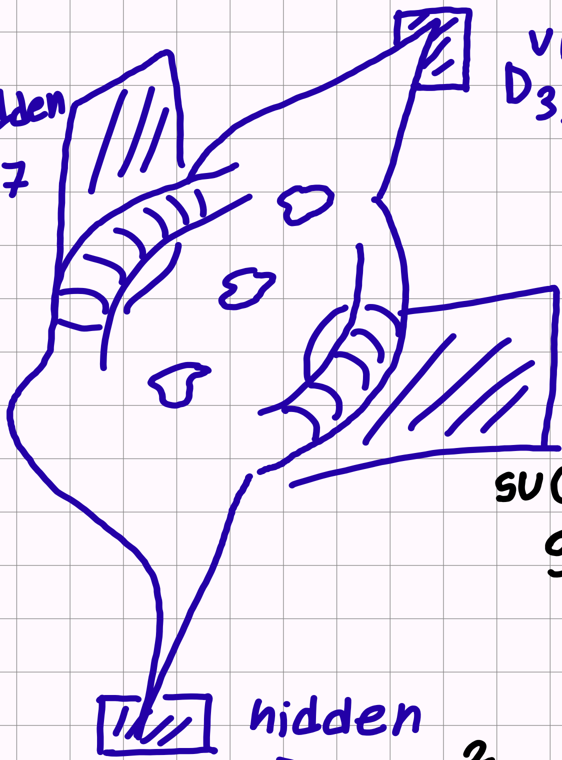
3) brane-world scenario

gauge interactions (open strings) are localized on branes while moduli (closed strings) propagate in bulk

$\Rightarrow$ physics decouples $\Rightarrow$ can study 2 problems seperately (at leading order)

- getting SM
- MOD STAB

# Pictorial view for Type IIB with D3 & D7




$g_{ym}^{-2} \sim Vol(\Sigma_4)$

$\mathcal{V}^{\frac{2}{3}}$

$g_{ym}^{-2} = \frac{1}{g_s}$

compute gauge coupling of an SU(N) TH on a stack of N Dp - branes

$$S_{DBI} = T_p \int_{W_{p+1}} d^{p+1} b \, e^{-4} \underbrace{\sqrt{-\det(g_{ab} + 2\pi\alpha' F_{ab})}}$$

$W_{p+1} = \mathbb{R}^{1,3} \times \Sigma_{p-3}$

$= \det(g^T + 2\pi\alpha' F^T)$

$= \det(g - 2\pi\alpha' F)$

$= [\det(1 + 2\pi\alpha' g^{-1} F) \det(1 - 2\pi\alpha' g^{-1} F)]^{1/4}$

$= [\det(1 - (2\pi\alpha')^2 (g^{-1}F)^2]^{1/4}$

$\sqrt{-\det(g_{ab})} \underbrace{\sqrt{\det(1 + 2\pi\alpha' g^{-1} F}}_{\parallel}$

$\sqrt{\det(1 - 2\pi\alpha' g^{-1} F}$

$$\ln \det A = \mathrm{Tr} \ln A$$
$$\det A = e^{\mathrm{Tr} \ln A}$$

$$\left[\det(1-(2\pi\alpha')^2(g^{-1}F)^2\right]^{\frac{1}{4}}$$
$$= e^{\frac{1}{4}\mathrm{Tr}\ln(1-(2\pi\alpha')^2(g^{-1}F)^2)}$$
$$= e^{-\frac{1}{4}\mathrm{Tr}((2\pi\alpha')^2(g^{-1}F)^2+\cdots)}$$
$$= 1-\frac{(2\pi\alpha')^2}{4}\underbrace{\mathrm{Tr}((g^{-1}F)^2)}_{\parallel}+\cdots\cdots$$

$$g^{ac}F_{cd}\,g^{db}F_{ba} = F^{ab}F_{ba}$$
$$= -F^{ab}F_{ab}$$

$$\Rightarrow S_{DBI} = -T_p\int_{W_{p+1}} d^{p+1}h\, e^{-4}\sqrt{-\det(g)}\left(1-\frac{(2\pi\alpha')^2}{4}F_{ab}F^{ab}+\cdots\right)$$

$$S_{DBI} \supset -T_p \mathrm{Vol}(\Sigma_{p-3})\, e^{-4}\frac{1}{4}(2\pi\alpha')^2\int d^4x\, F_{\mu\nu}F^{\mu\nu}$$
$$= -\frac{1}{4g_{YM}^2}\int d^4x\, F_{\mu\nu}F^{\mu\nu}$$

$$\Rightarrow g_{YM}^{-2} = \underbrace{T_p}_{\parallel\ (2\pi)M_s^{p+1}}\underbrace{(2\pi\alpha')^2}_{=\frac{1}{(2\pi)^2M_s^4}}\underbrace{\mathrm{Vol}(\Sigma_{p-3})}_{\parallel\ \tau_{p-3}M_s^{p-3}}e^{-\varphi} = \frac{\tau_{p-3}}{2\pi}e^{-\varphi}$$

made by nazlee (or nafisa)